Algebra 3 w/Trig
Review for Ch 9.1-9.4 TEST

Name: KEY

Simplify:

1) $\csc 38^\circ \cdot \sin 38^\circ$

$\frac{1}{\sin 38} \cdot \sin 38 = \boxed{1}$

2) $\sin 15^\circ \cos 65^\circ + \cos 15^\circ \sin 65^\circ$ $= \sin(15^\circ + 65^\circ) = \boxed{\sin 80^\circ}$

3) $\sec^2 x \cdot \sin x \cdot \cos x$

$\frac{1}{\cos^2 x} \cdot \sin x \cdot \cos x = \frac{\sin x}{\cos x} = \boxed{\tan x}$

4) $\frac{\tan 105^\circ + \tan 30^\circ}{1 - \tan 105^\circ \tan 30^\circ}$ $= \tan(105^\circ + 30^\circ) = \tan 135^\circ = \boxed{-1}$

Write each function in terns if its cofunction.

5) $\sin 35^\circ$

$\cos(90^\circ - 35^\circ)$

$\cos 55^\circ$

6) $\sin \frac{\pi}{6}$

$\cos\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$

$\cos\left(\frac{3\pi}{6} - \frac{\pi}{6}\right) = \boxed{\cos \frac{\pi}{3}}$

7) $\cot 50^\circ$

$= \tan(90^\circ - 50^\circ)$

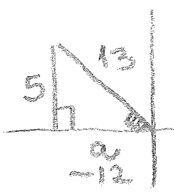
$= \tan 40^\circ$

8) $\cos 15^\circ$ $= \sin(90^\circ - 15^\circ)$

$= \sin 75^\circ$

Find the exact value of sine, cosine, and tangent of 2θ.

9. $\sin\theta = \frac{5}{13}, \theta$ in Quadrant II.



$a^2 + 5^2 = 13^2$

$a^2 = 169 - 25$

$a^2 = 144$

$a = \pm 12$

$a = -12$

$\boxed{\cos\theta = \frac{-12}{13}}$

(SOH-CAH-TOA)

$\sin 2\theta = 2\sin\theta\cos\theta = 2 \cdot \frac{5}{13} \cdot \left(\frac{-12}{13}\right) = \boxed{\frac{-120}{169}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120}{169} \Big/ \frac{119}{169} = \boxed{\frac{-120}{119}}$

10. $\cos\theta = -\frac{4}{5}, \theta$ in Quadrant II.

Find the exact value of sine, cosine and tangent of ½θ.

11. $\sin\theta = \frac{5}{13}, \theta$ in Quadrant II.

12. $\sin\theta = \frac{3}{5}, \theta$ in Quadrant I.

Find the exact value of the following using the addition or subtraction identities:

13. $\cos 75^\circ$

14. $\tan 15^\circ$

15. $\sin 105^\circ$

Simplify the following:

16. $\sin(x+\frac{\pi}{2})$

17. $\cos(x-\pi)$

18. If $\sin x = \frac{1}{4}$ and $0 < x < \frac{\pi}{2}$, then $\sin\left(\frac{\pi}{3}+x\right)=?$

19. If $\sin x = -\frac{2}{5}$ and $\frac{3\pi}{2} < x < 2\pi$, then $\cos\left(\frac{\pi}{4}+x\right)=?$

Prove the following identities:

20. $\dfrac{1}{\sec\theta-\tan\theta}=\sec\theta+\tan\theta$

21. $\cos^4\theta-\cos^6\theta=\cos^4\theta\cdot\sin^2\theta$

22. $\cos(x-\pi)=-\cos x$

22. $\cot x=\dfrac{\sin 2x}{1-\cos 2x}$

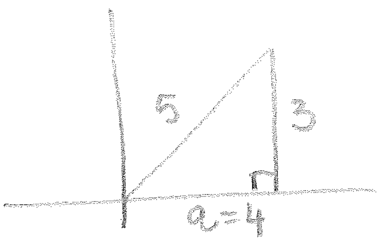
23. $\dfrac{\sin x}{\csc x}+\dfrac{\cos x}{\sec x}=1$

23. $(1+\cos x)(1-\cos x)=\sin^2 x$

$LS = (1+\cos x)(1-\cos x)$
$= 1-\cos^2 x$
$= \sin^2 x$
$= RS$

$\boxed{12}$

$\sin\theta = \frac{3}{5}$, θ in Quadrant I.



$3^2 + a^2 = 5^2$ $\quad \boxed{\cos\theta = 4/5}$

$9 + a^2 = 25$

$a^2 = 16$

$a = \pm 4$

$\boxed{a = 4}$

Since θ is in Quadrant I, $\frac{\theta}{2}$ is in quadrant I. Therefore, sine, cosine and tangent of $\theta/2$ are positive.

$$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-4/5}{2}} = \sqrt{\frac{\frac{5}{5}-\frac{4}{5}}{2}} = \sqrt{\frac{\frac{1}{5}}{2}} = \sqrt{\frac{1}{2}\cdot\frac{1}{5}} = \sqrt{\frac{1}{10}} = \frac{1}{\sqrt{10}} =$$

$$= \frac{1}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{\sqrt{10}}{10}}$$

$$\cos\frac{\theta}{2} = +\sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+4/5}{2}} = \sqrt{\frac{\frac{5}{5}+\frac{4}{5}}{2}} = \sqrt{\frac{\frac{9}{5}}{2}} = \sqrt{\frac{1}{2}\cdot\frac{9}{5}} = \sqrt{\frac{9}{10}} = \frac{3}{\sqrt{10}} =$$

$$= \frac{3}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \boxed{\frac{3\sqrt{10}}{10}}$$

$$\tan\frac{\theta}{2} = \frac{\sin\theta/2}{\cos\theta/2} = \frac{\sqrt{10}/10}{3\sqrt{10}/10} = \frac{\sqrt{10}}{3\sqrt{10}} = \boxed{\frac{1}{3}}$$

$\boxed{13}$

$$\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$
$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$\boxed{14}$

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = \frac{1 - \sqrt{3}/3}{1 + 1\cdot\frac{\sqrt{3}}{3}}$$

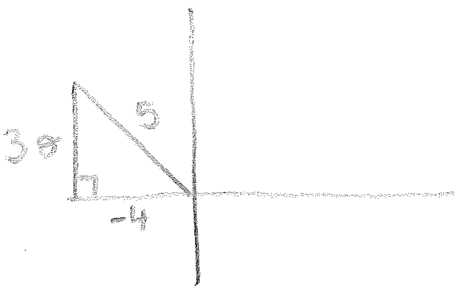
$$= \frac{\frac{3}{3} - \frac{\sqrt{3}}{3}}{\frac{3}{3} + \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{3}}{\frac{3+\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}\cdot\frac{3-\sqrt{3}}{3-\sqrt{3}} =$$

$$= \frac{9 - 3\sqrt{3} - 3\sqrt{3} + 3}{9 - 3} = \frac{12 - 6\sqrt{3}}{6} = \frac{12}{6} - \frac{6\sqrt{3}}{6} = \boxed{2 - \sqrt{3}}$$

$\boxed{15}$

$$\sin 105^\circ = \sin(60^\circ + 45^\circ) = \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} + \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$$

$\boxed{10}$ $\cos\theta = -\frac{4}{5}$, θ in Quadrant II



$\theta^2 + (-4)^2 = 5^2$

$\theta^2 = 25 - 16$

$\theta^2 = 9$

$\theta = \pm 3$

$\boxed{\theta = 3}$

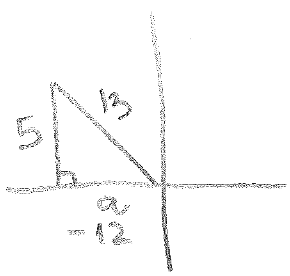
$\boxed{\sin\theta = \frac{3}{5}}$ (use SOH-CAH-TOA

$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \boxed{-\frac{24}{25}}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \boxed{\frac{7}{25}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-24/25}{7/25} = \boxed{-\frac{24}{7}}$

$\boxed{11}$ $\sin\theta = \frac{5}{13}$, θ in Quadrant II



$a^2 + 5^2 = 13^2$

$a^2 + 25 = 169$

$a^2 = 144$

$a = \pm 12$

$\boxed{a = -12}$

$\boxed{\cos\theta = -\frac{12}{13}}$

$\frac{\theta}{2}$ is in Quadrant I, therefore sine, cosine and tangent of ½θ are positive.

$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = +\sqrt{\frac{1-\left(-\frac{12}{13}\right)}{2}} = \sqrt{\frac{1+\frac{12}{13}}{2}} = \sqrt{\frac{\frac{13}{13}+\frac{12}{13}}{2}} = \sqrt{\frac{\frac{25}{13}}{2}} =$

$= \sqrt{\frac{1}{2}\cdot\frac{25}{13}} = \sqrt{\frac{25}{26}} = \frac{5}{\sqrt{26}} = \frac{5}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} \rightarrow \boxed{\frac{5\sqrt{26}}{26}}$

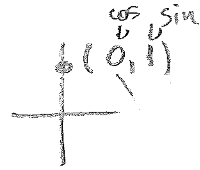
$\cos\frac{\theta}{2} = +\sqrt{\frac{1+\cos\theta}{2}} = \sqrt{\frac{1+\left(-\frac{12}{13}\right)}{2}} = \sqrt{\frac{1-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{13}{13}-\frac{12}{13}}{2}} = \sqrt{\frac{\frac{1}{13}}{2}} = \sqrt{\frac{1}{2}\cdot\frac{1}{13}}$

$= \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} = \frac{1}{\sqrt{26}}\cdot\frac{\sqrt{26}}{\sqrt{26}} = \boxed{\frac{\sqrt{26}}{26}}$

$\tan\frac{\theta}{2} = \frac{\sin\theta/2}{\cos\theta/2} = \frac{5\sqrt{26}}{26} \Big/ \frac{\sqrt{26}}{26} = \frac{5\sqrt{26}}{\sqrt{26}} = \boxed{5}$

16

$$\sin\left(x+\frac{\pi}{2}\right) = \sin x \cos\frac{\pi}{2} + \cos x \sin\frac{\pi}{2}$$



$$= \sin x \cdot 0 + \cos x \cdot 1$$

$$= 0 + \cos x$$

$$= \cos x$$

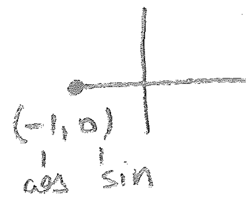
17

$$\cos(x-\pi) = \cos x \cos\pi + \sin x \sin\pi$$

$$= \cos x \cdot (-1) + \sin x \cdot (0)$$

$$= -\cos x + 0$$

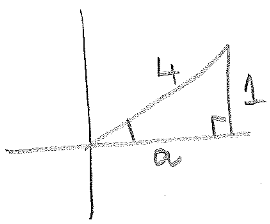
$$= \boxed{-\cos x}$$



18

$\sin x = \frac{1}{4}$, $0 < x < \frac{\pi}{2}$

Since $0 < x < \frac{\pi}{2}$, x is in Quadrant I.



$$a^2 + 1^2 = 4^2$$
$$a^2 + 1 = 16$$
$$a^2 = 15$$
$$a = \sqrt{15}$$

$$\boxed{\cos x = \frac{\sqrt{15}}{4}}$$

$$\sin\left(\frac{\pi}{3}+x\right) = \sin\frac{\pi}{3}\cos x + \sin x \cos\frac{\pi}{3}$$

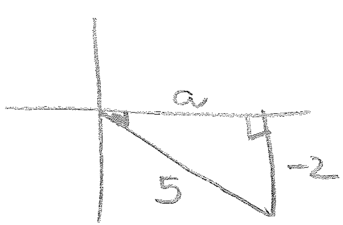
$$= \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{15}}{4} + \frac{1}{4}\cdot\frac{1}{2} = \frac{\sqrt{45}}{8} + \frac{1}{8} = \frac{\sqrt{45}+1}{8} = \boxed{\frac{3\sqrt{5}+1}{8}}$$

$$\sqrt{45} = \sqrt{9\cdot 5} = \sqrt{9}\cdot\sqrt{5} = 3\sqrt{5}$$

19

$\sin x = -\frac{2}{5}$ and $\frac{3\pi}{2} < x < 2\pi$

Since $\frac{3\pi}{2} < x < 2\pi$, x is in Quadrant IV



$$a^2 + (-2)^2 = 5^2$$
$$a^2 + 4 = 25$$
$$a^2 = 21$$
$$a = \pm\sqrt{21}$$
$$\boxed{a = \sqrt{21}}$$

$$\boxed{\cos x = \frac{\sqrt{21}}{5}}$$

$$\cos\left(\frac{\pi}{4}+x\right) = \cos\frac{\pi}{4}\cos x - \sin\frac{\pi}{4}\sin x$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{21}}{5} - \frac{\sqrt{2}}{2}\cdot\left(\frac{-2}{5}\right)$$

$$= \frac{\sqrt{42}}{10} + \frac{2\sqrt{2}}{10} = \boxed{\frac{\sqrt{42}+2\sqrt{2}}{10}}$$

$\boxed{\#20}$ $\dfrac{1}{\sec\theta-\tan\theta} = \sec\theta+\tan\theta$

$$LS = \frac{1}{\sec\theta - \tan\theta} = \frac{1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}} = \frac{1}{\frac{1-\sin\theta}{\cos\theta}} = \frac{\cos\theta}{1-\sin\theta} = \frac{\cos\theta}{1-\sin\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{\cos\theta(1+\sin\theta)}{1-\sin^2\theta} = \frac{\cancel{\cos\theta}(1+\sin\theta)}{\cos^{\cancel{2}}\theta} = \frac{1+\sin\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta = RS \checkmark$$

$\boxed{\#21}$ $\cos^4\theta - \cos^6\theta = \cos^4\theta\sin^2\theta$

$$L\$ = \cos^4\theta - \cos^6\theta \underset{\substack{\uparrow \\ \text{Factor}}}{=} \cos^4\theta(1-\cos^2\theta) = \cos^4\theta\cdot\sin^2\theta = R\$ \checkmark$$

$\boxed{\#22}$ $\cos(x-\pi) = -\cos x$

$$L\$ = \cos(x-\pi) = \cos x\cos\pi + \sin x\sin\pi = \cos x(-1) + \sin x\cdot(0)$$

$$= -\cos x + 0 = -\cos x = R\$ \checkmark$$

$\boxed{\#22}$ $\cot x = \dfrac{\sin 2x}{1-\cos 2x}$

$$R\$ = \frac{\sin 2x}{1-\cos 2x} = \frac{2\sin x\cos x}{1-(\cos^2 x-\sin^2 x)} = \frac{2\sin x\cos x}{\underbrace{1-\cos^2 x}_{\sin^2 x}+\sin^2 x} = \frac{2\sin x\cos x}{\sin^2 x+\sin^2 x}$$

$$= \frac{\cancel{2}\cancel{\sin x}\cos x}{\cancel{2}\sin^{\cancel{2}} x} \underset{\substack{\uparrow \\ \text{reduce} \\ \text{fraction}}}{=} \frac{\cos x}{\sin x} = \cot x = L\$ \checkmark$$

$\boxed{\#23}$ $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

$$L\$ = \frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = \frac{\sin x}{\frac{1}{\sin x}} + \frac{\cos x}{\frac{1}{\cos x}} = \sin x\cdot\frac{\sin x}{1} + \cos x\cdot\frac{\cos x}{1} = \sin^2 x + \cos^2 x$$

$$= 1 = RS \checkmark$$