Homework check:

- Have all your homework from previous chapter- solving trigonometric equation ready for check.
- Using only your notes and homework work independently to solve the equations .
- These class assignment will count as a **quiz .** It is worth **20 points.**

Homework check:

- Using **only your notes and homework** work independently to solve the equations bellow. **Find exact solutions if possible**.

1. Find all solutions of $\cos 2x + \frac{\sqrt{2}}{2} = 0.$

2. Find solutions in the interval $[0,2\pi)$

$$2\sin^2 x + \sin x - 2 = 0$$

Learning Targets

- Apply various strategies to proof identities

Identities and Proofs

- **Identity** is an equation that is true for all values of the variable for which every term of the equation is defined.

- **Trigonometric identities** can be used for simplifying expressions, rewriting the rules of trigonometric functions, and performing numerical calculations

Solving equations vs. Proving Identities

- Find all the numbers that make the equation true.
- Ex.

$$\tan x \cos^2 x = \tan x$$

Solution:

$$x = k\pi, \qquad k \in \mathbb{Z}$$

- Show than an equation is true for all numbers.(for which all terms of the equation are defined
- Ex.

$$\tan x = \frac{\sin x}{\cos x}$$

Solution:

The equation is true for all values of x such that

$$\cos x \neq 0$$

Proving Identities

- A useful feature of the trigonometric functions is that they can be written in many ways. One form may be easier to use in one situation , and a different form of the same function may be more useful in another.
- The elementary identities used in Chapter 6 have to be memorized.
- Memorizing these identities will benefit you greatly in the future.
- Let's review the basic identities…

Basic Identities

- **Quotient Identities**

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x}$$

- **Reciprocal Identities**

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x}$$

$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

Basic Identities

- **Periodicity Identities**

$$\sin(x \pm 2\pi) = \sin x \qquad \cos(x \pm 2\pi) = \cos x$$

$$\csc(x \pm 2\pi) = \csc x \qquad \sec(x \pm 2\pi) = \sec x$$

$$\tan(x \pm \pi) = \tan x \qquad \cot(x \pm \pi) = \cot x$$

- **Negative Angle Identities**

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \tan(-x) = -\tan x$$

Basic Identities

- **Pythagorean Identities**

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad 1 + \cot^2 x = \csc^2 x$$

$$\sin^2 x = 1 - \cos^2 x \qquad \tan^2 x = \sec^2 x - 1 \qquad \cot^2 x = \csc^2 x - 1$$

$$\cos^2 x = 1 - \sin^2 x$$

Strategies for proving Trigonometric Identities

1. Use algebra and *previously* proven identities to transform one side of the equation into the other.
2. If possible, write the entire equation in terms of one trigonometric function.
3. Express everything in terms of sine and cosine.
4. Deal separately with each side of the equation $A = B$. First use identities and algebra to transform A into some expression C, then use (possibly different) identities and algebra to transform B into the *same* expression C. Conclude that $A = B$.

There are often a variety of ways to proceed, and it will take some practice before you can easily decide which strategies are likely to be the most efficient in a particular case. Keep these two purposes of working with trigonometric identities in mind:

- to learn the relationships among the trigonometric functions
- to simplify an expression by using an equivalent form

CAUTION

Proving identities is not the same as solving equations. Properties that apply to equations, such as adding the same value to both sides, are not valid when verifying identities because the beginning statement (to be verified) may not be true.

Transform one side into the other side

strategy used in Chapter 6

Verify that $\dfrac{1 + \sin x - \sin^2 x}{\cos x} = \cos x + \tan x.$

$$\frac{1 + \sin x - \sin^2 x}{\cos x} = \frac{(1 - \sin^2 x) + \sin x}{\cos x}$$ *regrouping terms*

$$= \frac{\cos^2 x + \sin x}{\cos x}$$ *Pythagorean identity*

$$= \frac{\cos^2 x}{\cos x} + \frac{\sin x}{\cos x}$$ $\dfrac{a+b}{c} = \dfrac{a}{c} + \dfrac{b}{c}$

$$= \cos x + \frac{\sin x}{\cos x}$$ $\dfrac{a^2}{a} = a$

$$= \cos x + \tan x$$ *quotient identity*

More Examples:

Prove that $\frac{\cos x \tan x}{\sin x} = 1.$

Solution

$$\text{L.S.} = \frac{\cos x \tan x}{\sin x}$$

Use the quotient identity: $= \frac{\cos x \sin x}{\sin x \cos x}$

Divide by common factors: $= 1$

$$\text{R.S.} = 1$$

L.S. = R.S., so $\frac{\cos x \tan x}{\sin x} = 1.$

One for you to try:

Prove that $\cos x = \dfrac{1}{\cos x} - \sin x \tan x.$

Solution

$$\text{R.S.} = \frac{1}{\cos x} - \sin x \tan x$$

Use the quotient identity: $= \frac{1}{\cos x} - \frac{\sin x \sin x}{\cos x}$

Simplify: $= \frac{1 - \sin^2 x}{\cos x}$

Use the Pythagorean identity: $= \frac{\cos^2 x}{\cos x}$

Divide by the common factor: $= \cos x$

$$\text{L.S.} = \cos x$$

L.S. = R.S., so $\cos x = \frac{1}{\cos x} - \sin x \tan x$.

Daily Practice:

- Worksheet- proofs day 1

#1-10

Graphical Identity Testing

- Any equation can be tested by simultaneously graphing the function given by the left and right sides of the equation.
- If the graphs are different the equation is not a identity.
- If the graphs appear to be the same, it is possible that the equation is an identity.
- **Caution: graphical testing does not prove the equation is an identity.**

Graphical Identity Testing

- Is the equation an identity?

$$2\sin^2 x - \cos x = 2\cos^2 x + \sin x$$

Graph each side of the equation on the same screen.

$$Y_1 = 2\sin^2 x - \cos x$$

$$Y_2 = 2\cos^2 x + \sin x$$

The graphs are different, therefore the equation is not an identity.

Is the equation an identity?

$$\frac{1+\sin x-\sin^2 x}{\cos x}=\cos x+\tan x$$

Graph each side of the equation on the same screen

$$Y_1=\frac{1+\sin x-\sin^2 x}{\cos x}$$

$$Y_2=\cos x+\tan x$$

CAUTION

Be sure to use parentheses correctly when entering each function to be graphed.

Graphical Identity Testing

- Some for you to try:

A) $$\frac{\sec x - \cos x}{\sec x} = \sin^2 x$$

B) $\tan x + \cot x = \sin x \cos x$