

Agenda for Today

- Revisit coterminal angles and finding the lowest angle coterminal with a given angle. (10 min)
- Checking point(10 min).
- Introduce new key terms: arc length, central angle. (15 min)
- Daily practice. (15 min)

Learning Targets

- Know the definition of the key terms:
 - ▣ coterminal angles
 - ▣ Arc length and central angle
- Understand how to find the lowest coterminal angle, arc length, and central angle.

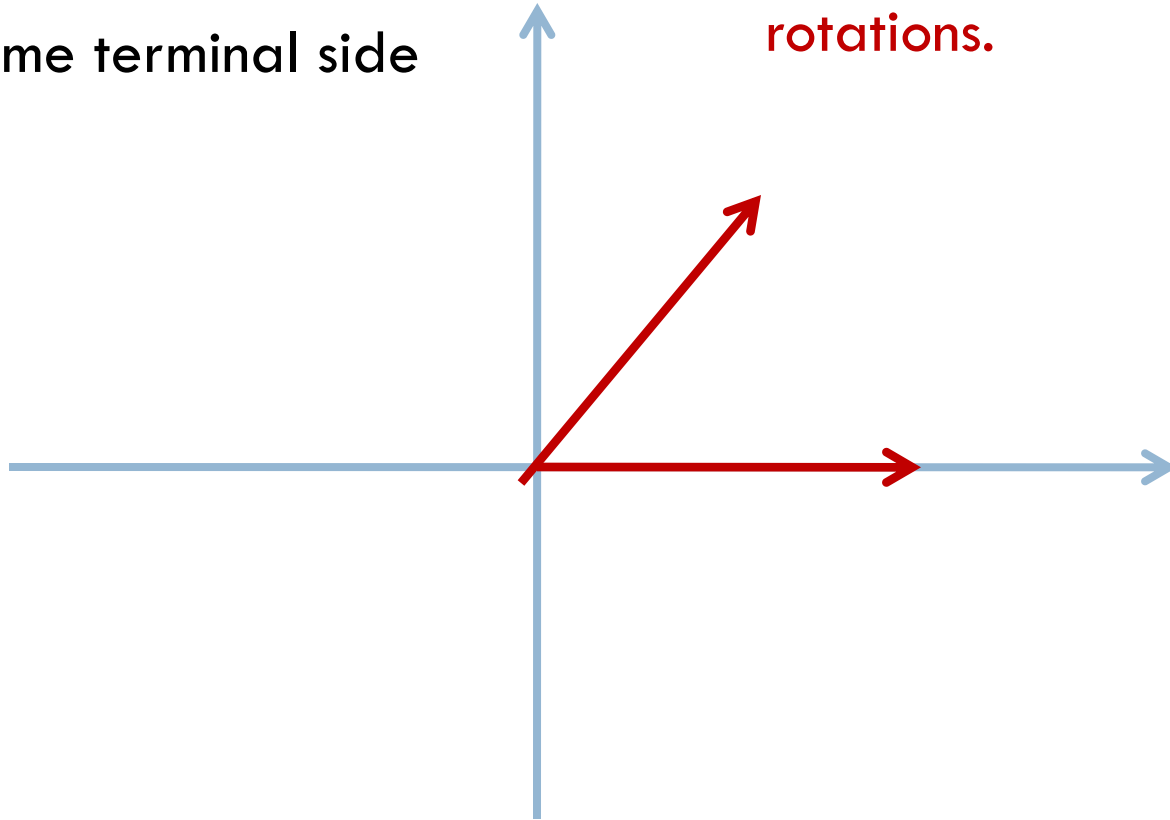
Coterminal angles

Coterminal Angles

Are two different angles

With the same terminal side

To find Coterminal angles
add or subtract complete rotations.



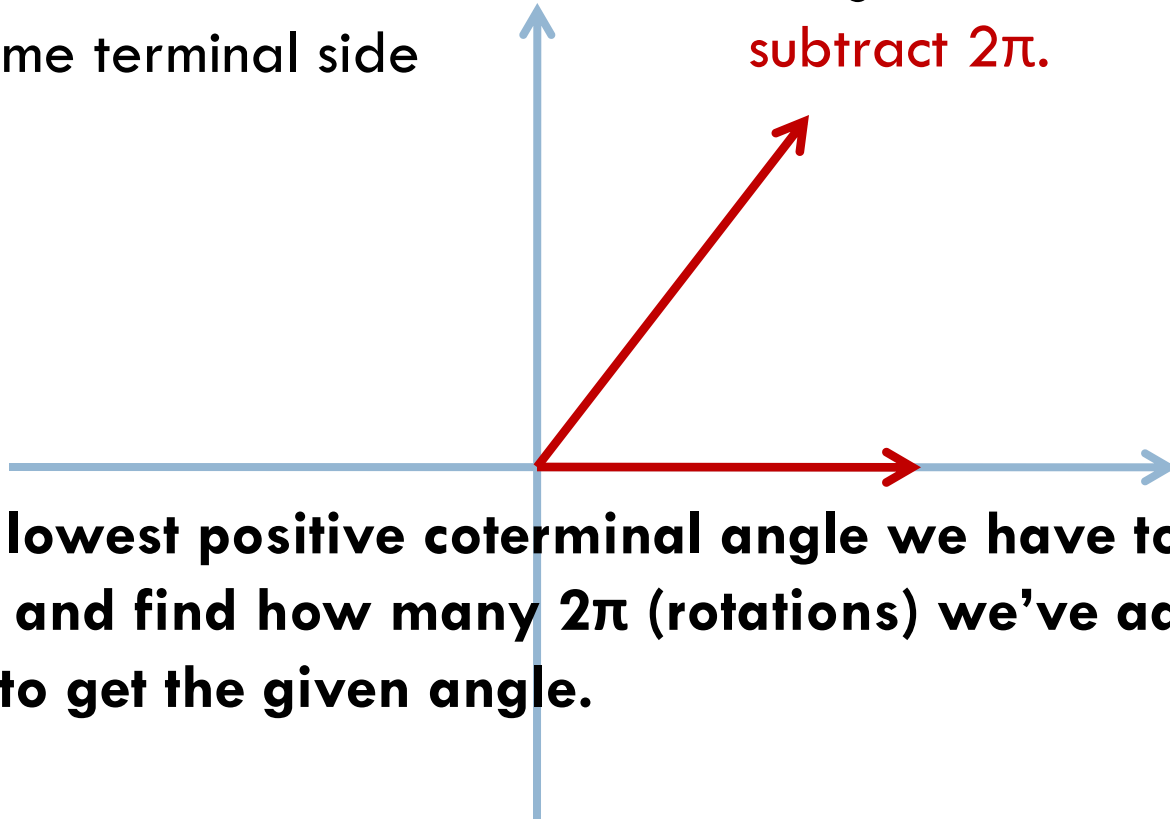
Coterminal angles

Coterminal Angles

Are two different angles

With the same terminal side

To find Coterminal angles using radians **add or subtract 2π** .



To find the lowest positive coterminal angle we have to work backwards and find how many 2π (rotations) we've added or subtracted to get the given angle.

Coterminal angles

To find the lowest positive coterminal angle we have to work backwards and find how many 2π (rotations) we've added or subtracted to get the given angle.

- Find an angle in radians , between 0 and 2π , that is coterminal with

$$\frac{7\pi}{3}$$

Coterminal angles

- Find an angle in radians, between 0 and 2π , that is coterminal with

$$\frac{7\pi}{3}$$

Solution:

$$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3}$$

$$= 2\pi + \frac{\pi}{3}$$

The positive coterminal angle is $\frac{\pi}{3}$

Coterminal angles

- Some for you to try:
- Find the lowest positive angle, in radians, coterminal with the given angle.

1. $\frac{21\pi}{5}$

2. $-\frac{8\pi}{3}$

3. 8

4. -9

Coterminal angles

□ Answers

$$1. \quad \frac{\pi}{5} \qquad \frac{21\pi}{5} = \frac{20\pi}{5} + \frac{\pi}{5} = 4\pi + \frac{\pi}{5}$$

$$2. \quad \frac{4\pi}{3}, \qquad -\frac{8\pi}{3} = -\frac{6\pi}{3} - \frac{2\pi}{3} = -2\pi - \frac{2\pi}{3}$$

A coterminal angle is $-\frac{2\pi}{3}$, but is not positive

Change to positive:

$$2\pi - \frac{2\pi}{3} = \frac{6\pi}{3} - \frac{2\pi}{3} = \frac{4\pi}{3}.$$

Answer $\frac{4\pi}{3}$.

Coterminal angles

□ Answers

3. $8 - 2\pi$

Since $2\pi \approx 6.28$, to change 8 to a number between 0 and 2π , we have to subtract 2π . The answer is $8 - 2\pi$.

4. $-9 + 4\pi$

For -9 we have to add multiples of 2π .

$$-9 + 2\pi \approx -2.72$$

$$-9 + 4\pi \approx 3.56$$

Checking point

- **Do you know what coterminal angles are and how to find the lowest positive angle in radians coterminal with a given angle?**
- ✓ I fully understand the concept. 😊
- + I understand the concept, but I need more practice. 😐
- ? I am lost. I have no clue. 😞

Checking point

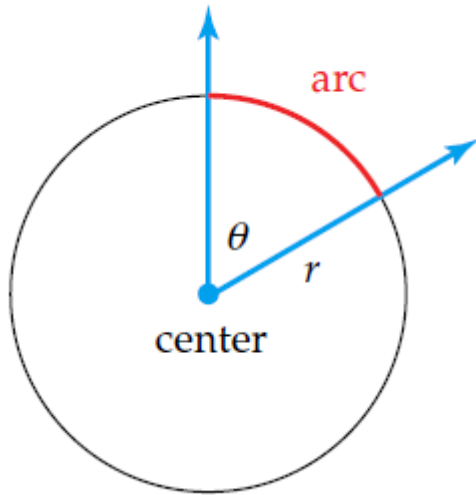
Find the angle in radians between 0 and 2π that is coterminal with the given angles.

a) $\frac{23\pi}{6}$

b) $-\frac{7\pi}{2}$



Arc Length and Central Angle



Arc = a part of a circle
Central angle = angle whose vertex is the center of the circle

Arc length- ℓ depends on the central angle θ , and the radius, r .

$$\frac{360}{\theta} = \frac{2\pi r}{\ell}$$

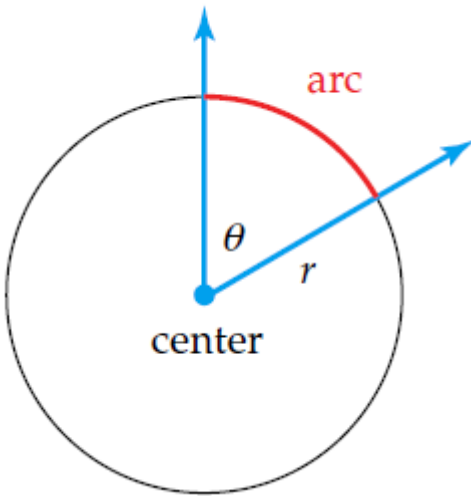
$$\ell = \frac{\theta}{360} \cdot 2\pi r = \frac{\theta\pi r}{180}$$

Arc Length and Central Angle

- Using radian measure the formula for the arc length simplifies to

$$\ell = r\theta$$

where r = the radius of the circle
 θ = central angle in radians

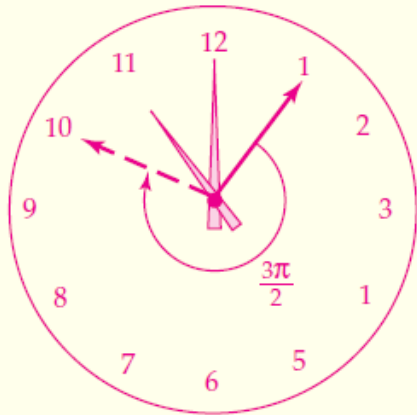


Finding Arc Length

$$\ell = r\theta$$

EXAMPLE:

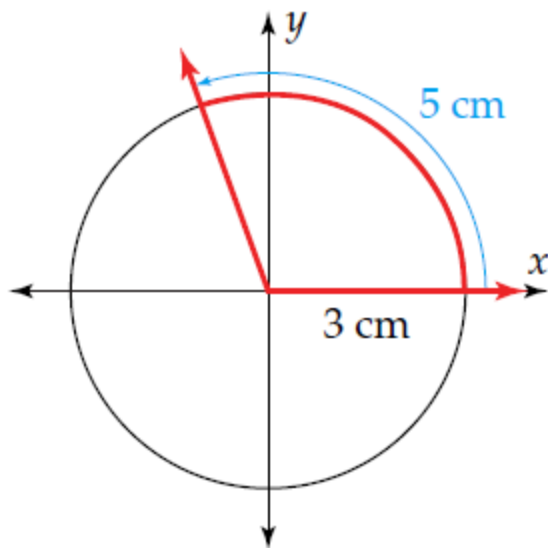
The second hand on a clock is 5 inches long. How far does the tip of the second hand move in 45 seconds?



$$5\left(\frac{3\pi}{2}\right) \approx 23.6 \text{ inches}$$

Finding Central Angle Measure

Find the central angle measure (in radians) of an arc of length 5 cm on a circle with a radius of 3 cm.



$$\ell = r\theta$$

$$\ell = 5, r = 3$$

$$5 = 3\theta$$

$$\theta = \frac{5}{3} \text{ rad}$$

Daily Practice

1. Determine the positive radian measure of the angle that the second hand of a clock travels through in 2 minutes 25 seconds.
2. The second hand on a clock is 5 cm long. How far does the tip travel in 40 seconds?
3. Assume that the wheel on a car has a radius of 40 cm. Find the angle (in radians) that the wheel turns while the car travels 510 cm.

Daily Practice – answers:

1. $29\pi/6$
2. $20\pi/3$
3. $51/4 \text{ rad} = 12.75 \text{ rad}$

Summary

- Coterminal angle
- Finding coterminal angles
- Definitions :Arc , Central angle
- Formula for arc length
- Formula for central angle

Homework

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