Limit Theorem.

Let x be any real number, k be any real constant, n be a natural number , then as $|x| \to \infty$,

$$\frac{1}{x} \to 0, \qquad \frac{1}{x^n} \to 0, \qquad \frac{k}{x^n} \to 0$$

Find the limits

Ex.1 $\lim_{x\to\infty} x^3 = \infty$

Ex 2. $\lim_{x\to\infty}(x^2+3) = \lim_{x\to\infty}(x^2) = \infty$

EX.3 $\lim_{x\to\infty} \frac{x}{2x+1} = \lim_{x\to\infty} \frac{x}{x(2+\frac{1}{x})} = \frac{1}{2}$

Finding limits of rational Functions

Step 1. Keep only the leading terms for each polynomial

Step 2. Reduce the fraction.

Step 3. Find the limit:

If the fraction reduces to a real number, the number is your limit.

If the fraction has an x left in the denominator, the limit is zero.

If the fraction has an x left in the numerator, the limit is ∞

Example:

$$\lim_{x \to \infty} \frac{9x - 6}{2x + 6} = \lim_{x \to \infty} \frac{9x}{2x} = \frac{9}{2}$$

$$\lim_{x \to \infty} \frac{x + 3}{x^3 - 9} = \lim_{x \to \infty} \frac{x}{x^3} = \lim_{x \to \infty} \frac{1}{x^2} = 0$$

$$\lim_{x \to \infty} \frac{2x^2 - 3x + 1}{x - 3} = \lim_{x \to \infty} \frac{2x^2}{x} = \lim_{x \to \infty} (2x) = \infty$$