Fundamental Theorem of Algebra

If $P(x)$ is a polynomial of degree $n \geq 1$ $with$ complex coefficients, then $P(x) = 0$ has at least one complex root.

Number of Roots Theorem

If $P(x)$ is a polynomial of degree $n \geq 1$ $with$ complex coefficients, then $P(x) = 0$ has exactly n roots.

Ex 1 Find a polynomial $f(x)$ of degree 4 such that $-3, -1$ and 2 are zeros, 2 is of multiplicity 2 and $f(1) = 32$.

$$f(x) = a(x+3)(x+1)(x-2)^2$$

$$\boxed{f(x) = 4(x+3)(x+1)(x-2)^2}$$

$$32 = a(1+3)(1+1)(1-2)^2$$

$$32 = a \cdot 4 \cdot 2 \cdot 1$$

$$32 = 8a$$

$$a = 4$$

EX 2

Find the zeros of $f(x) = x^2 - 4x + 5$. *then* write f(x) as a product of linear factors.

$$x^2 - 4x + 5 = 0$$

$$x = \frac{4 \pm \sqrt{16-20}}{2}$$

$$x = \frac{4 \pm 2i}{2}$$

$$\sqrt{-4}$$

$$2i$$

$$\boxed{x = 2 \pm i}$$

Root	Factor
$2+i$	$x-2-i$
$2-i$	$x-2+i$

$$x = 2+i$$

$$-2-i \quad -2-i$$

$$x-2-i = 0$$

$$\boxed{f(x) = (x-2-i)(x-2+i)}$$

Ex 3

Find a polynomial with real coefficients whose zeros include the numbers – 3 *and* $2-i$. , $2+i$

* If $a+bi$ is a zero, then $a-bi$ is a zero.

$$(x+3)(x-2+i)(x-2-i)$$

$$(x+3)(x^2-2x-\cancel{xi}-2x+4+\cancel{2i}+\cancel{ix}-\cancel{2i}-i^2)$$

-1

$$(x+3)(x^2-4x+5)$$

Find all the zeros given one zero

a) $x^4 - 6x^2 + 25, 2 + i$, $2-i$

b) $x^4 - 4x^3 + 3x^2 + 8x - 10 = 0, 2 + i$

$$\begin{array}{r|rrrrr} 2+i & 1 & 0 & -6 & 0 & 25 \\ & \downarrow & 2+i & 3+4i & -10+5i & -25 \\ \hline 2-i & 1 & 2+i & -3+4i & -10+5i & \boxed{0} \\ & \downarrow & 2-i & 8-4i & 10-5i & \\ \hline & 1 & 4 & 5 & \boxed{0} & \end{array}$$

$x^2 + 4x + 5 = 0$

$x = \frac{-4 \pm \sqrt{-4}}{2} = -2 \pm i$

$(2+i)(2+i)$

$4 + 4i + i^2$

$(2+i)(-3+4i)$

$-6 + 5i + 4i^2$
-4

$(2+i)(-10+5i)$

$-20 + \cancel{10i} - \cancel{10i} + 5i^2$

$-20 - 5 = -25$

$x^4-4x^3+3x^2+8x-10=0$, $2+i$, $2-i$

$(x-2-i)(x-2+i)$

$= x^2-2x+\cancel{ix}-2x+4-\cancel{2i}-\cancel{ix}+\cancel{2i}-i^2$

$i^2 = -1$

$= x^2-4x+5$

$x^2-2=0$

$x^2=2$

$x=\pm\sqrt{2}$

$$
\begin{array}{r|l}
 & x^2-2 \\
x^2-4x+5 & x^4-4x^3+3x^2+8x-10 \\
 & -x^4+4x^3+5x^2 \\
\hline
 & -2x^2+8x-10 \\
(-) & -2x^2+8x-10 \\
\hline
 & \boxed{0}
\end{array}
$$

Hw:

p. 313 # 11, 13,15, 23,27, 31, 49-55(odds)