Complex numbers

i = imaginary unit

$i^2 = -1$ or $\sqrt{-1} = i$

Complex numbers

$a + bi$; $a - bi$ (conjugate)

real part, imaginary part

Eg. $2 - 3i$ & $2 + 3i$

$1 + 5i$ $\quad$ $1 - 5i$

1) Add.

$(2+3i) + (5+6i)$

$(2+5) + (3i+6i)$

$7 + 9i$

2) Subtract

$(2+3i) - (5+2i)$

$-3 + 1i$

$-3 + i$

Multiplication

$(2+3i)(3-2i)$

FOIL

$6 - 4i + 9i - 6i^2$

$12 + 5i$

$i^2 = -1$

$+6$

$(2+3i)(2-3i)$

$4 - 6i + 6i - 9i^2$

$4 + 9$

13

$(a+bi)(a-bi) = a^2 + b^2$

$(3-2i)(3+2i) = 3^2 + 2^2 = 9 + 4 = 13$

Division

$$\frac{2-3i}{1+2i} \cdot \frac{1-2i}{1-2i}$$

$$\frac{2-4i-3i+6\underbrace{i^2}_{-1}}{1^2+2^2}$$

$$\frac{-4-7i}{5}$$

$$\boxed{-\frac{4}{5} - \frac{7}{5}i}$$

$$\frac{1}{4i} \cdot \frac{i}{i}$$

$$\frac{i}{4i^2}$$

$$\frac{1 \cdot i}{-4}$$

$$= \underbrace{-\frac{1}{4}i}$$

$$\frac{2-3i}{1+i} \cdot \frac{1-i}{2+3i}$$

$$\frac{2-2i-3i\underbrace{+3i^2}_{-1}}{2+3i+2i\underbrace{+3i^2}_{-3}}$$

$$\frac{-1-5i}{-1+5i} \cdot \frac{-1-5i}{-1-5i}$$

$$= \frac{1+5i+5i+25i^2}{1+5^2}$$

$$= \frac{-24+10i}{26} = \underbrace{-\frac{24}{26} + \frac{10}{26}i}$$

$$\frac{3-2i}{1+i} \cdot \frac{1-2i}{2-3i}$$

$$\sqrt{-16} = 4i$$

$$\sqrt{-8} = i\sqrt{8} = 2i\sqrt{2}$$

$$\sqrt{-12} \cdot \sqrt{-3} = \cancel{\sqrt{36} = 6}$$

$$i\sqrt{12} \cdot i\sqrt{3} = i^2\sqrt{36}$$

$$= \boxed{-6}$$

$\sqrt{-16} = 4i$

$\sqrt{-8} = i\sqrt{8}$

$\frac{\sqrt{-16}}{\sqrt{-4}} = \frac{4i}{2i} = 2$

$\sqrt{-1} = i$

$\sqrt{i} = i^{1/2}$

$\sqrt{-12} \cdot \sqrt{-4} = i\sqrt{12} \cdot 2i$ (−1)

$= -2\sqrt{12}$

$= -4\sqrt{3}$

$i^0 = 1$ (Remainder 0)

$i^1 = i$ (1)

$i^2 = -1$ (2)

$i^3 = -i$ (3)

$i^4 = 1$

$i^5 = i$

$i^6 = -1$

$i^7 = -i$

$i^8 = 1$

$i^{102} = i^2 = -1$

25
$4 \overline{)102}$
8
22
20
2

$i^{100} = i^0 = 1$

$i^{29} = i^1 = i$

$29 \div 4 = 7$ (r. 1)

$8 - 2yi = 4x + 12i$

$8 = 4x$

$2 = x$

$-2y = 12$

$y = -6$

<56>

p 300 #1-58 (odds)