Zeros with multiplicity k

Example: *Find the zeros of the polynomial and state the multiplicity of each zero.*

A) $f(x) = (x-1)^3(x+3)^2(x-2)$

B) $f(x) = (x^3-4x)^2$

A) Zeros:

1- Multiplicity 3

-3 -Multiplicity 2

2 - Multiplicity 1

B) $(x^3-4x)^2 = [x(x^2-4)]^2$

$$= x[(x-2)(x+2)]^2$$

$$= x^2(x-2)^2(x+2)^2$$

Zeros : 0-multiplicity 2,

2 – multiplicity 2,

-2 –multiplicity 2

Decarte's Rule of Signs

Let $p(x)$ define a polynomial function with real coefficients written in descending powers of x. Count the number of sign changes in the signs of the coefficients.

1. *The number of positive real zeros is equal to the number of sign changes in $p(x)$ or is equal to that number decreased by an even integer.*
2. *The number of negative real zeros is equal to the number of sign change in $p(x)$ or is equal to that number decreased by an even integer.*

$$p(x) = 2x^4 - 9x^3 - 8x^2 + 29x - 10$$
$$p(-x) = 2x^4 + 9x^3 - 8x^2 - 29x - 10$$

R+	R-	I
3	1	0
1	1	2

$$f(x) = 5x^3 - 2x^4 + x^2 - 7$$
$$f(x) = -2x^4 + 5x^3 + x^2 - 7$$
$$f(-x) = -2x^4 - 5x^3 + x^2 - 7$$

R+	R-	I
2	2	0
0	0	4

2 0 2

0 2 2

Possible rational roots

Rational Roots Theorem

If $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients and p/q is a rational zero, then $p\ is\ a\ factor\ of\ a_0$ and $q\ is\ a\ factor\ of\ a_n$.

List all the possible rational roots of

A) $x^3 - 2x^2 - 4x + 4 = 0$ B) $2x^3 + x^2 - 13x + 6 = 0$

A) $p = 4, factors: \pm 1, \pm 2, \pm 4$

$q = 1$ factors 1

$p/q: \pm {}^1/_1, \pm \frac{2}{1}, \pm \frac{4}{1}$

Possible roots: $\pm 1, \pm 2, \pm 4$

B) $p = 6$, factors: $1, \pm 2, \pm 3, \pm 6$

$q = 2$, factors $1, 2$

$p/q: \pm {}^1/_1, \pm \frac{2}{1}, \pm \frac{3}{1}, \pm \frac{6}{1}, \pm \frac{1}{2}, \pm \frac{2}{2}, \pm \frac{3}{2}, \pm \frac{6}{2}$

Possible roots: $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

Daily Practice

<47> WS #1-35 odds