

Polynomial Function

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_2 x^2 + a_1 x + a_0$$

Where n is an integer greater or equal to zero and coefficients are real numbers.

How to recognize a polynomial?

- All exponents are whole numbers
- No variables in denominator
- No variable under radical

a_n = leading coefficient

a_0 = constant term

In Exercises 1–8, determine whether the given algebraic expression is a polynomial. If it is, list its leading coefficient, constant term, and degree.

① $1 + x^3$

② -7

③ $(x - 1)(x^2 + 1)$

~~④ $7^x + 2x + 1$~~

⑤ $(x + \sqrt{3})(x - \sqrt{3})$

~~⑥ $4x^2 + 3\sqrt{x} + 5$~~

~~⑦ $\frac{7}{x^2} + \frac{5}{x} - 15$~~

⑧ $(x - 1)^k$ (where k is a fixed positive integer)

Name	Degree	Example
Constant	0	$P(x) = 6$
Linear	1	$P(x) = 3x + 4$
Quadratic	2	$P(x) = -2x^2 + 3x + 9$
Cubic	3	$P(x) = 2x^3 + x^2 + x + 1$
Quartic	4	$P(x) = 3x^4 + 2x^3 - 11x + 4$

Long Division

$$(6x^3 - 19x^2 + x + 6) \div (x - 3)$$

$$\begin{array}{r} 6x^2 - x - 2 \\ x-3 \overline{) 6x^3 - 19x^2 + x + 6} \\ \underline{-6x^3 + 18x^2} \\ -x^2 + x \\ \underline{+x^2 + 3x} \\ -2x + 6 \\ \underline{+2x + 6} \\ \boxed{0} \end{array}$$

$$\begin{aligned} \frac{6x^3}{x} &= 6x^2 \\ 6x^2(x-3) &= \\ &6x^3 - 18x^2 \end{aligned}$$

Synthetic Division

- can only be used when dividing by $x - r$ (r can be + or -)

Example :

$$\begin{array}{r} 6x^3 - 19x^2 + x + 6 \\ \hline x - 3 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 6 & -19 & +1 & +6 \\ & \downarrow & 18 & -3 & -6 \\ \hline & 6 & -1 & -2 & \boxed{0} \end{array}$$

quotient
 $6x^2 - x - 2$

remainder

0

$$\begin{array}{r} -5x^5 - 21x^4 - 3x^3 + 4x^2 - 2x + 2 \\ \hline x + 4 \end{array}$$

$$\begin{array}{r|rrrrrrr} -4 & -5 & -21 & -3 & 4 & -2 & 2 \\ & \downarrow & 20 & 4 & -4 & 0 & 8 \\ \hline & -5 & -1 & 1 & 0 & -2 & \boxed{10} \end{array}$$

quotient:
 $-5x^4 - x^3 + x^2 - 2$

remainder: 10

$$\begin{array}{r} x^3 - 27 \\ \hline x - 3 \end{array}$$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & \downarrow & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & \boxed{0} \end{array}$$

$x^2 + 3x + 9$

<46> p. 248 # 9-22